

The calculation of  $U_0$  in the case of  $^3\text{He}$  is complicated by the fact that ordering of the nuclear spins starts in the *liquid phase* below roughly  $1^\circ\text{K}$ . We have tried to eliminate this effect by extrapolating the various quantities from above  $1^\circ\text{K}$  towards  $0^\circ\text{K}$ . The thermal energy associated with complete spin ordering is, however, certainly less than 1 cal/mole and is therefore not important for the present purpose.

From the vapour pressure data of Sydoriak & Roberts (1957) we find  $L_0 = -5.04$  cal/mole for liquid  $^3\text{He}$  at zero pressure. The work of compression of the liquid up to the melting pressure was estimated from the data of Sherman & Edeskuty (1960); this gives 2.5 cal/mole. The change in  $U_0$  upon solidification was estimated as 0.8 cal/mole from the results of Mills, Grilly & Sydoriak (1961), and of Grilly & Mills (1959). The melting pressure at  $0^\circ\text{K}$  was taken as 29 Kg/cm<sup>2</sup>. Table 10 gives the different steps of the calculation. The work of compression up to  $V = 16.5$  cm<sup>3</sup>/mole, the lowest part of the  $0^\circ\text{K}$  isotherm known experimentally was estimated at 20.5 cal/mole. For this we used, as in the case of  $^4\text{He}$ , an extrapolation of the  $0^\circ\text{K}$  isotherm taking the melting line of Grilly & Mills (1959) as a guide. This procedure is reliable to probably better than 1 cal/mole because the  $0^\circ\text{K}$  isotherm is quite close to the melting line in this density range. The transition, from b.c.c.  $^3\text{He}$  to h.c.p.  $^3\text{He}$ , which was estimated by Mills & Grilly (1959) to have zero volume change at  $0^\circ\text{K}$ , has been neglected.

Values of  $U_0$  for both isotopes are shown as a function of volume in figure 13.

#### 4. DISCUSSION

##### 4.1. *The specific heat of solid helium*

At temperatures sufficiently low that the wavelengths of the excited lattice vibrations are long compared to the interatomic distance, the continuum model should describe the behaviour of the actual solid very accurately and  $C_v$  should be given by the well-known relation

$$C_v = \frac{12\pi^4}{5} R \left( \frac{T}{\theta_0} \right)^3. \quad (11)$$

Here  $R$  is the gas constant and  $\theta_0$  the limiting low-temperature value of the Debye temperature which can also be calculated from the low-temperature elastic constants of the solid (cf. Barron & Klein 1962). This  $T^3$  behaviour is to be expected only at temperatures below about  $\theta/50$ . Above this  $\theta_D$  will in general be temperature dependent; figure 14 illustrates the kind of temperature dependence to be expected of  $\theta_D$  for a harmonic solid with short-range forces and a close-packed structure. At high temperatures  $\theta_D$  would, in this model, again become constant at a value usually referred to as  $\theta_\infty$ .

For solid  $^3\text{He}$  and  $^4\text{He}$  we see (from figures 7 and 8) the fall in  $\theta_D$  with increasing temperature but no minimum in the curve. This may be because melting and 'pre-melting' (perhaps the onset of vacancy formation) prevent us from seeing this part of the curve at these densities. The magnitude of the change in  $\theta_D$  with temperature is, however, not very different from that found in other inert gas solids such as argon and krypton over the corresponding range of temperatures (cf. Beaumont, Chihara & Morrison 1961).

If at these densities solid helium approximated to a harmonic solid, the values of  $\theta_D$  at lower temperatures would be similar to those indicated by the dashed lines in figures 7 and 8.